AD-A039 554 NAVY UNDERWATER SOUND LAB NEW LONDON CONN F/G 12/1

WEIGHTED LINEAR REGRESSION FOR TWO VARIABLES.(U)

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UNCLASSIFIED NUSL-TM-2070-451-69 NL

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NUSL Problem No
A-403-00-00
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WEIGHTED LINEAR REGRESSION FOR TWO VARIABLES

by

John Skory and Robert C. Jennings

NUSL Technical Memorandum No 2070-451-69
NUSL-TM-

11 December 1969

51p.

INTRODUCTION

The primary purpose of linear regression is the prediction of Y from a given value of X by means of the regression equation Y=BX or Y=A+BX , and the estimation of confidence limits for true Y , A and B . Regression analysis for two variables may be made on a weighted or unweighted basis with the line passing through the origin or through the means.

NON-CONSTANT VARIANCE OF Y AND WEIGHTING

If Y is the dependent variable, the linear regression of Y on X may be calculated if X is measured without error and each single measurement of Y has the same variance over the whole range of X . When this is not the case deviations from the regression line will not be normally distributed and the prediction of Y from X will be suspect.

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When the variance requirement is not met, a transformation of Y to logarithms or square root, etc., may be tried whereby the variances at the lower end of the scale of X will be found to be non-significantly different from those at the upper. If repeat measurements of Y at one value of X are available, the reciprocal of the variance derived from such a cluster may be used as a weight in the analysis. If variable Y comprises both single measurements and averaged measurements, the number of measurements that were averaged should be used as a weight. If both the variances of Y and number of measurements per averaged Y are known the weights would be the number of measurements of Y at X_i divided by the variance of Y at X_i. If the variance of Y at X_i is a function of X then X_i^{-2} may be used as a weight.

PROGRAM EXTENSIONS

Program REGRESS (S1281) was developed and written by J. Skory and Z. Ungar at the United Aircraft Corporate Systems Center for the IBM 1800. It has been adapted to the UNIVAC 1108 with the following modifications and additions:

(1) the maximum input sample size has been increased to 2000 observations.

(2) the weighted mean $\bar{X}$, weighted corrected sum of squares SSX, and the weighted corrected sum of products SSXY, are now evaluated by the recursion formulas:

$$\bar{X}_k = \frac{\sum^{k-1} W_i}{\sum^{k} W_i} \bar{X}_{k-1} + \frac{W_k X_k}{\sum^{k} W_i}$$

$$SSX_k = SSX_{k-1} + \frac{\sum^{k-1} W_i}{\sum^{k} W_i} W_k (X_k - \bar{X}_{k-1})^2$$

$$SSXY_k = SSXY_{k-1} + \frac{\sum^{k-1} W_i}{\sum^{k} W_i} W_k (X_k - \bar{X}_{k-1}) (Y_k - \bar{Y}_{k-1})$$

with k interated from 1 to n for a sample of n

(3) Confidence limits for true Y are calculated at the experimental X_i and post experimental X_i .

(4) Effective N at X_i is calculated for use in finding tolerance limits for Y . Effective N is a value such that average Y at a given X is predicted as accurately from the point on the regression line as if N observations had been actually made at that specified X .

(5) Linear calibration formulas are given for prediction of X from Y with confidence limits for true X .

(6) Formulas are given for the statistics necessary for the comparison of 2 regression lines. Tests are provided for homogeneity of variances about the lines, parallelism and coincidence.

(The last three items are calculated by the user after the program has been run.)

INPUT INFORMATION

The program reads all the following input data from the card reader (unit 3): (For I/O formats see Appendix A).

(1) n the number of data points in the input sample.

(2) IH an indicator, = 1 for regression through the origin (Y = BX) or = 2 for regression through the means (Y =A+BX).

(3) IW an indicator, = 1 if all weights = 1 , or = 2 if $W_i = X_i^{-2}$, or = 3 if weights are given.

(4) t Student's t for calculating confidence limits for true Y .

(5) $X_1 \ldots X_n$ the independent variable at n points equally or unequally spaced along the X axis and assumed to be controlled by the experimenter without error (or only with negligible error).

(6) $Y_1 \ldots Y_n$ the dependent variable. Y_i is an associate of X_i and if Y is an individual single measurement $\mathrm{Var}(Y_1) = \mathrm{Var}(Y_2) = \ldots = \mathrm{Var}(Y_n)$.

(7) $W_1 \ldots W_n$ weights for Y_i given as input only if IH = 3.

(8) $P_1 \ldots P_{10}$ ten points of X chosen post experimentally at which it is desired to predict Y and estimate its error as an exploratory step for future experimentation.

(9) $Q_1 \ldots Q_{10}$ ten weights associated with P_i .

COMPUTER ANALYSIS

Program REGRESS first sorts the points of the independent variable X into ascending order using a modified Shell sort embodied in SØRT, a Fortran subroutine (NUSL Program 1525) programmed by Lesick and Shores and modified to retain the Y_i and W_i associated with each X_i . This is done in order to obtain the within groups sums of squares for use in a one-way analysis of variance.

The program then evaluates the equation Y=A+BX or Y=BX where A and B are weighted least squares estimates determined by minimizing the sums $\sum W_i(Y_i-\hat{Y}_i)^2$ or $\sum W_i(Z_i-\hat{Z}_i)^2$ where the Z_i are translates of Y_i . The program thus performs a weighted linear regression analysis through the <u>means</u> or through the <u>origin</u>.

The deviations of observed Y from the regression line are tested for normality using WTEST , a Fortran function (NUSL Program S1640), (References 2 and 3).

The program writes all output data on the line printer (Unit 4):

SUMWX and SUMWY	sum of the weighted X_i and Y_i
SUMW	sum of the weights
AVERX, AVERY	weighted average of X_i and Y_i
SUMDX SUMDY SUMXY	weighted sum of squares and products about the mean or origin
B	the regression coefficient

A	the intercept
$\hat{Y}_i$	predicted Y vector
$\hat{Y}_{ei}$	predicted exploratory Y vector
$(Y_i-\hat{Y}_i)$	vector of deviations of observed Y from the regression line
R^2	correlation coefficient squared
S	standard error of estimate
F	a variance ratio testing if $B=R=0$ $F(1,n-1)$, through the origin, is evaluated $F(1,n-2)$, through the means, is evaluated
S_B	standard error of B
S_A	standard error of A
F	another variance ratio testing if $A=0$ (for case of through the means). $F(1,n-2)$ is evaulated with 1 and $(n-2)$ degrees of freedom
$S_{\hat{Y}_i}$	standard error of each <u>predicted</u> point on the line at X_i
$S_{\hat{Y}_{ei}}$	standard error of each <u>predicted</u> exploratory (postulated) point on the line at post experimental $X_i (=P_i)$
S_{Y_i}	standard error of each weighted newly observed Y or $(Y_i-\hat{Y}_i)$

$S_{Y_{ei}}$	standard error of each weighted newly observed Y_{ei} or $(Y_{ei}-\hat{Y}_{ei})$
t	Student's t for each deviation from regression:
	t(n-1) for regression through the origin
	t(n-2) for regression through the means
VARY	variance of Y within groups at one X
SSB	sum of squares due to B
SSDEV	sum of squares of deviations from the line
U	normal distribution test statistic (applicable only when all W_i are equal)
CL	confidence limits for true Y at experimental and post experimental X_i
N	effective N

The mathematical theory and derivation is presented, for example, in K.A. Brownlee (1960). In the calculations, many of the statistics are evaluated by alternate algebraically equivalent methods.

POST EXPERIMENTAL ESTIMATION

After an experiment is completed one may wish to estimate Y at some points of X(=P) not already tried in the experiment and obtain confidence limits for these estimates. In addition, one may wish to know how many repeat measurements, Q , are required at a specified value of X(=P) so that the error of a weighted newly observed Y is

less than θ at 95% confidence. This may be done by making $P_1=P_2=P_3$ and varying Q_1, Q_2, Q_3 widely and plotting the error against Q.

POST COMPUTER-ANALYSIS CALCULATIONS

From the computer output further calculations may be made by desk calculator to obtain the following five types of estimated information.

(1) Tolerance Limits

Using effective N in the manner of (4) (page 498) the tolerance limits for Y may be estimated. The following is the type of statement that would result; There is 95% confidence that, at the X specified, 99% of the population of individual measurements (observations) of Y , in future experimentation, will fall between the limits calculated.

(a) Tolerance limits at experimental X_i equals

$\hat{Y}_i \pm (ru)_i S$ (Using $\hat{N}_i$)

(b) Tolerance limits at post experimental X_i

$\hat{Y}_{ei} \pm (ru)_{ei} S$ (Using $\hat{N}_{ei}$)

(c) Tolerance limits for W_i newly observed Y at experimental X_i

$\hat{Y}_i \pm (ru)_i S$ (Using N_i)

(d) Tolerance limits for Q_i newly observed Y at post experimental X_i

$\hat{Y}_{ei} \pm (ru)_{ei} S$ (Using N_{ei})

(2) Linear Calibration: Prediction of X from Y

A. Sample Unweighted: Line Through the Means

(1) Confidence limits for true X at a weighted newly observed Y

$$\bar{X} + \frac{Y-\bar{Y}}{B} \pm tF^{-.5}\left(\left(\frac{1}{W}+\frac{1}{n}\right)\sum_i^n (X_i-\bar{X})^2 + \frac{(Y-\bar{Y})^2}{B^2}\right)^{1/2} = X$$

where t = Student's t for (n-2) degrees of freedom

F comes from test of significance for B.

If $F < 1$ results may not be meaningful.

W is the number of Y's averaged to give the new observation Y.

(2) Standard error of $X = \frac{\text{upper limit-lower limit}}{2t}$

B. Sample Unweighted: Line Through the Origin

(1) Confidence limits for true X at a weighted newly observed Y

$$X = \frac{Y}{B} \pm tF^{-.5}\left(\frac{\sum^n X_i^2}{W} + \frac{Y^2}{B^2}\right)^{1/2}$$

with t at (n-1) degrees of freedom.

(2) Standard error of $X = \frac{\text{upper limit-lower limit}}{2t}$

C. Sample Weighted: Line Through the Means

(1) Confidence limits for true X at a weighted newly observed Y are obtained by solving for X in the quadratic:

$$\left(Y-\bar{Y}-B(X-\bar{X})\right)^2 = \frac{t^2B^2}{F}\left(\left(\frac{X^2}{W}+\frac{1}{\sum^n W_i}\right)\sum^n W_i(X_i-\bar{X})^2 + (X-\bar{X})^2\right)$$

(2) Standard error of $X = \frac{\text{upper limit-lower limit}}{2t}$

D. Sample Weighted: Line Through the Origin

(1) Confidence limits for true X at a weighted newly observed Y

$$X = \frac{Y}{B \pm tS\left(\frac{1}{W}+\frac{1}{n}\right)^{\frac{1}{2}}}$$

(2) Standard error of $X = \dfrac{\text{upper limit-lower limit}}{2t}$

(3) Comparison of Two Regression Lines ($W_i=1$)

Given the estimates:

Line 1: $\bar{X}_1$ $\bar{Y}_1$ B_1 n_1 S_1^2 f_1 $\Sigma(X-\bar{X}_1)^2$ $\Sigma(Y-\bar{Y}_1)^2$

Line 2: $\bar{X}_2$ $\bar{Y}_2$ B_2 n_2 S_2^2 f_2 $\Sigma(X-\bar{X}_2)^2$ $\Sigma(Y-\bar{Y}_2)^2$

A. Test If $S_1^2 = S_2^2$

Make 2-sided F test with larger S^2 in the numerator.

If S_1^2 is larger:

$$F = \frac{S_1^2}{S_2^2} \quad \text{with } (f_1, f_2)$$

B. Test If $B_1 = B_2$ (A Test of Parallelism)

If S_1^2 is not significantly different from S_2^2 they are pooled and

$$S^2 = \frac{f_1 S_1^2 + f_2 S_2^2}{f_1 + f_2}$$

Then

$$\frac{B_1 - B_2}{S\left(\frac{1}{\Sigma(X-\bar{X}_1)^2} + \frac{1}{\Sigma(X-\bar{X}_2)^2}\right)^{\frac{1}{2}}} = t(f_1+f_2)$$

C. Test If the Two Lines Are Coincidental (if $\bar{X}_1=\bar{X}_2$; $\bar{Y}_1=\bar{Y}_2$; given $B_1 = B_2$)

Pooled estimate of B

$$\frac{B_1 \sum^{n_1}(X_i-\bar{X}_1)^2 + B_2 \sum^{n_2}(X_i-\bar{X}_2)^2}{\sum^{n_1}(X_i-\bar{X}_1)^2 + \sum^{n_2}(X_i-\bar{X}_2)^2} = B$$

Pooled variance of estimate = S^2 =

$$\frac{\sum^{n_1}(Y_i-\bar{Y}_1)^2 + \sum^{n_2}(Y_i-\bar{Y}_2)^2 - B^2\left(\sum^{n_1}(X_i-\bar{X}_1)^2 + \sum^{n_2}(X_i-\bar{X}_2)^2\right)}{f_1+f_2+1}$$

Then

$$\frac{\bar{Y}_1-\bar{Y}_2-B(\bar{X}_1-\bar{X}_2)}{S\left(\frac{1}{n_1}+\frac{1}{n_2}+\frac{(\bar{X}_1-\bar{X}_2)^2}{\sum^{n_1}(X_i-\bar{X}_1)^2+\sum^{n_2}(X_i-\bar{X}_2)^2}\right)^{1/2}} = t(f_1+f_2+1)$$

(4) Analysis of Variance

The computer output contains the necessary information to perform a one-way analysis of variance.

(5) Simultaneous Tolerance Intervals

These intervals are calculated as $\hat{Y}_{ei} \pm c^{**}S_{\hat{Y}_{ei}}$ with c** tabled and its use explained in 4.7.4 of Reference 5.

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4. Weissberg, A., and Beatty, G.H., 1960. Tables of Tolerance-Limit Factors for Normal Distributions. Technometrics Vol 2, No. 4.

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APPENDIX A

NOMENCLATURE TABLE

Fortran Variable Name & Type	Meaning
X real array	The independent variable of the input.
Y real array	The dependent variable of the input.
W real array	The vector of weights associated with the independent variable X .
P real array	The vector of post-experimental points of the independent variable X .
Q real array	The vector of weights associated with P .
N integer	The number of observations in the input variables X and Y .
IH integer	An indicator for regression through the means or through the origin.
IW integer	An indicator for the weights W .
STUDNT real	Student's t for calculation of confidence levels.
TITLE real array	An array used to store the Hollerith characters of the title card.
SUMW double precision	The sum of the weights W .
SUMWX double precision	The weighted sum of X .
SUMWX2 double precision	The weighted sum of X^2 .
SUMWY double precision	The weighted sum of Y .

NOMENCLATURE TABLE (Cont'd)

Fortran Variable Name & Type	Meaning
SUMWY2 double precision	The weighted sum of Y^2 .
SUMWXY double precision	The weighted sum of XY .
AVERX double precision	The weighted mean of X .
AVERY double precision	The weighted mean of Y .
SUMDX double precision	The weighted corrected sum of squares of X .
SUMDY double precision	The weighted corrected sum of squares of Y .
SUMXY double precision	The weighted corrected sum of products of X and Y .
B double precision	The regression coefficient.
A double precision	The regression intercept.
YHAT real array	The predicted Y of regression.
REGRY real array	The deviations of Y from the regression line.
SUMRS double precision	The weighted sum of squares of the deviations.
RSQD double precision	The multiple and simple correlation coefficient squared.
SSB double precision	The weighted sum of squares due to B .
S double precision	The standard error of estimate.
AW1 double precision	Multiplier for intercept variance.
SYHAT real array	The standard error of each predicted point.
SY real array	The standard error of each predicted point assuming these points are newly observed Y with weights = W .

NOMENCLATURE TABLE (Cont'd)

Fortran Variable Name & Type	Meaning
T real array	Student's t for each deviation from the regression line.
CLY1H real array	The upper confidence limits for true Y at each experimental X_i .
CLY1L real array	The lower confidence limits for true Y at each experimental X_i .
CLY2H real array	The upper confidence limits for true Y at each experimental X_i assuming these points are newly observed Y with weights = W .
CLY2L real array	The lower confidence limits for true Y at each experimental X_i assuming these points are newly observed Y with weights = W .
NHAT real array	Effective N for tolerance limits at each experimental X_i .
NI real array	Effective N for tolerance limits at each experimental X_i assuming these points are newly observed Y with weights = W .
YSUBE real array	The predicted exploratory Y after regression.
SYHE real array	The standard error of each predicted exploratory point.
SEYE real array	The standard error of each predicted exploratory point assuming these points are newly observed Y_e with weights = Q .
CLYE1H real array	The upper confidence limits for true Y at each post-experimental X_{ei} .
CLYE1L real array	The lower confidence limits for true Y at each post-experimental X_{ei} .

NOMENCLATURE TABLE (Cont'd)

Fortran Variable Name & Type	Meaning
CLYE2H real array	The upper confidence limits for true Y at each post-experimental X_{ei} assuming these points are newly observed Y_e with weights = Q .
CLYE2L real array	The lower confidence limits for true Y at each post-experimental X_{ei} assuming these points are newly observed Y_e with weights = Q .
NEHAT real array	Effective N for tolerance limits for each post-experimental X_{ei} .
NE real array	Effective N for tolerance limits for each post-experimental X_{ei} assuming these points are new Y_e with weights = Q .
F12B double precision	The F test for R = B = 0 .
SB double precision	The standard error of the regression coefficient.
SA double precision	The standard error of the regression intercept.
F12A	The F test for A = 0 .
U real	W-test for normality of the deviations.
SUMN integer	The degrees of freedom within groups.
SUMSQ double precision	The sum of squares within groups.
NGRØUP integer	The size of each group.
SSY double precision	The variance of each group.
YBAR double precision	The mean of each group.

APPENDIX B

I/O FORMATS

The program reads (TITLE(1), 1 = 1,14) from the first card in 13A6, AZ format.

The program reads N,IH,IW, and STUDNT from the second card:

Variable	Columns	Format
N	1-5	I5
IH	6-10	I5
IW	11-15	I5
STUDNT	16-30	E15.8

The program then reads the input variables X,Y,W (only if IW = 3), P, and Q separately from cards in 5E15.8 format.

The program writes all output data on the line printer in 7E17.8 format.

APPENDIX C

```
C      REGRESS--PROGRAM FOR TWO-VARIABLE WEIGHTED REGRESSION (THROUGH
C      THE ORIGIN AND THROUGH THE MEANS)
       DIMENSION TITLE(14), X(2000), Y(2000), W(2000), YHAT(2000),
      1     YSUBE(2000), REGRY(2000), SYHAT(2000), SY(2000), T(2000),
      2     P(10), Q(10), SEYE(10), SYHE(10)
       DIMENSION CLY1H(2000), CLY1L(2000), CLYE1H(2000), CLYE1L(2000),
      1     CLY2H(2000), CLY2L(2000), CLYE2H(2000), CLYE2L(2000),
      2     NHAT(2000), NEHAT(2000), NI(2000), NE(2000)
       REAL NHAT,NI,NEHAT,NE
       INTEGER SUMN, DEGFB1, DEGFC
       DOUBLE PRECISION SUMW, SUMWX, SUMWX2, SUMWY, SUMWY2, SUMXY, SUMWXY
      1     , SUMDX, SUMDY, SUMRS, RSQD, S, SA, SB, A, B, AVERX, AVERY,
      2     F12A, F12B, AW1, S2, SSB, SUMY, SSY, SUMSQ, YBAR, MNSQB,
      3     MNSQB1, MNSQB2, MNSQC, B1, F2, C, BGSMSQ, SMGRPY
  1000 READ (5,700,END=999) (TITLE(I),I=1,14)
       READ(5,5)N,IH,IW,STUDNT
       READ(5,6)(X(I),I=1,N)
       READ(5,6)(Y(I),I=1,N)
       IF ( IW .EQ. 3 )  READ(5,6) (W(I),I=1,N)
       READ(5,6)(P(I),I=1,10)
       READ(5,6)(Q(I),I=1,10)
       GO TO (15,16,17), IW
    15 DO 18 I=1,N
       W(I) = 1.0
    18 CONTINUE
       GO TO 17
    16 DO 19 I=1,N
```

```
      W(I) = X(I)**(-2)
   19 CONTINUE
   17 CALL SORT3(X,Y,W,1,N,.TRUE.)
      IF ( IH .EQ. 2 )  GO TO 27
      WRITE(4,67)
      GO TO 28
   27 WRITE(4,7)
   28 WRITE(4,800)(TITLE(I),I=1,14)
      WRITE(4,12)N,IH,IW,STUDNT
      WRITE (4,23)
      WRITE (4,50) (X(I),I=1,N)
      WRITE(4,8)
      WRITE(4,50)(Y(I),I=1,N)
      WRITE(4,24)
      WRITE(4,50)(W(I),I=1,N)
      WRITE(4,21)
      WRITE(4,50)(P(I),I=1,10)
      WRITE(4,25)
      WRITE(4,50)(Q(I),I=1,10)
      SUMW = 0.0D0
      SUMWX = 0.0D0
      SUMWX2 = 0.0D0
      SUMWY = 0.0D0
      SUMWY2 = 0.0D0
      SUMWXY = 0.0D0
      AVERX = 0.0D0
      AVERY = 0.0D0
      SUMDX = 0.0D0
      SUMDY = 0.0D0
```

```
      SUMXY = 0.0D0
      IF ( IN .EQ. 2 )  GO TO 31
      DO 29 I=1,N
      SUMWX=SUMWX+W(I)*X(I)
      SUMWX2 = SUMWX2 + W(I)*X(I)*X(I)
      SUMWY=SUMWY+W(I)*Y(I)
      SUMWY2 = SUMWY2 + W(I)*Y(I)*Y(I)
      SUMWXY = SUMWXY + W(I)*X(I)*Y(I)
      SUMW=SUMW+W(I)
   29 CONTINUE
      SUMDX = SUMWX2
      SUMDY = SUMWY2
      SUMXY = SUMWXY
      GO TO 32
   31 DO 26 I=1,N
      SUMDX = SUMDX + SUMW/(SUMW+W(I)) * W(I) * (X(I)-AVERX)**2
      SUMDY = SUMDY + SUMW/(SUMW+W(I)) * W(I) * (Y(I)-AVERY)**2
      SUMXY = SUMXY + SUMW/(SUMW+W(I)) * W(I)*(X(I)-AVERX)*(Y(I)-AVERY)
      AVERX = SUMW/(SUMW+W(I)) * AVERX  +  W(I)*X(I)/(SUMW+W(I))
      AVERY = SUMW/(SUMW+W(I)) * AVERY  +  W(I)*Y(I)/(SUMW+W(I))
      SUMWX=SUMWX+W(I)*X(I)
      SUMWX2 = SUMWX2 + W(I)*X(I)*X(I)
      SUMWY=SUMWY+W(I)*Y(I)
      SUMWY2 = SUMWY2 + W(I)*Y(I)*Y(I)
      SUMWXY = SUMWXY + W(I)*X(I)*Y(I)
      SUMW=SUMW+W(I)
   26 CONTINUE
   32 B=SUMXY/SUMDX
      A=AVERY-B*AVERX
```

```
      SUMRS = 0.0D0
      DO 13 I=1,N
      YHAT(I)=A+B*X(I)
      REGRY(I) = Y(I) - YHAT(I)
      SUMRS = SUMRS + W(I)*(Y(I)-YHAT(I))**2
   13 CONTINUE
      RSQD=(SUMDY-SUMRS)/SUMDY
      SSD=RSQD*SUMDY
      S2=(1.0D0-RSQD)*SUMDY/(N-IH)
      S=SQRT(S2)
      AW1=1.0/SUMW
      IF ( IH .EQ. 1 )  AW1 = 0.0D0
      DO 41 I=1,N
      SYHAT(I) = S * SQRT( AW1 + (X(I)-AVERX)**2/SUMDX )
      SY(I) = S * SQRT( 1./W(I) + AW1 + (X(I)-AVERX)**2/SUMDX )
      T(I)=REGRY(I)/SY(I)
      CLY1H(I) = YHAT(I) + STUDNT*SYHAT(I)
      CLY1L(I) = YHAT(I) - STUDNT*SYHAT(I)
      CLY2H(I) = YHAT(I) + STUDNT*SY(I)
      CLY2L(I) = YHAT(I) - STUDNT*SY(I)
      NHAT(I) = S2 / SYHAT(I)**2
      NI(I) = S2 / SY(I)**2
   41 CONTINUE
      DO 46 I=1,10
      YSUBE(I)=A+B*P(I)
      SYHE(I) = S * SQRT( AW1 + (P(I)-AVERX)**2/SUMDX )
      SEYE(I) = S * SQRT( 1./G(I) + AW1 + (P(I)-AVERX)**2/SUMDX )
      CLYE1H(I) = YSUBE(I) + STUDNT*SYHE(I)
      CLYE1L(I) = YSUBE(I) - STUDNT*SYHE(I)
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```
      CLYE2H(I) = YSUBE(I) + STUDNT*SEYE(I)
      CLYE2L(I) = YSUBE(I) - STUDNT*SEYE(I)
      NEHAT(I) = S2 / SYHE(I)**2
      NE(I) = S2 / SEYE(I)**2
   46 CONTINUE
      F12B=(N-IH)*RSQD/(1.0-RSQD)
      SB=ABS(B)/SQRT(F12B)
      SA = S * SQRT( AW1 + AVERX**2/SUMDX )
      IF ( IH .EQ. 2 )  F12A = A**2/SA**2
      WRITE(4,102)SUMWX,SUMWY,SUMW
      WRITE(4,103)AVERX,AVERY
      WRITE(4,104)SUMDX,SUMDY
      WRITE(4,105)SUMXY
      WRITE(4,106)B,SSB
      WRITE(4,107)A
      WRITE(4,128)A,B
      WRITE(4,108)
      WRITE(4,50)(YHAT(I),I=1,N)
      WRITE(4,109)
      WRITE(4,50)(YSUBE(I),I=1,10)
      WRITE(4,110)
      WRITE(4,50)(REGRY(I),I=1,N)
      WRITE(4,111)RSQD
      WRITE(4,112)S,SUMRS
      WRITE(4,113)IH,F12B
      WRITE(4,114)SB
      WRITE(4,115)SA
      U = WTEST(REGRY,N,.FALSE.)
      IF(IH-2)144,147,147
```

```
  147 WRITE(4,116)IH,F12A
  144 WRITE(4,117)
      WRITE(4,50)(SYHAT(I),I=1,N)
      WRITE(4,118)
      WRITE(4,50)(SYHE(I),I=1,10)
      WRITE(4,119)
      WRITE(4,50)(SY(I),I=1,N)
      WRITE(4,125)
      WRITE(4,50)(SEYE(I),I=1,10)
      WRITE(4,121)IH
      WRITE(4,51)(T(I),I=1,N)
      WRITE(4,122) U
      WRITE(4,123)
      WRITE(4,53)(CLY1H(I),CLY1L(I),I=1,N)
      WRITE(4,124)
      WRITE(4,53)(CLYE1H(I),CLYE1L(I),I=1,10)
      WRITE(4,126)
      WRITE(4,53)(CLY2H(I),CLY2L(I),I=1,N)
      WRITE(4,127)
      WRITE(4,53)(CLYE2H(I),CLYE2L(I),I=1,10)
      WRITE(4,130)
      WRITE(4,50)(NHAT(I),I=1,N)
      WRITE(4,131)
      WRITE(4,50)(NEHAT(I),I=1,10)
      WRITE(4,132)
      WRITE(4,50)(NI(I),I=1,N)
      WRITE(4,133)
      WRITE(4,50)(NE(I),I=1,10)
      WRITE(4,134)
```

```
      SUMN = 0
      SUMSQ = 0.0
      NUMGRP = 0
      SMGRPY = 0.0D0
      BGSMSQ = 0.0D0
      I = 1
   39 NGROUP = 1
      ISTART = 1
   36 I = I+1
      IF (I .LE. N)  GO TO 35
      IF (NGROUP .GT. 1)  GO TO 40
      GO TO 42
   35 IF (X(ISTART)-X(I)) 34, ,34
      NGROUP = NGROUP+1
      GO TO 36
   34 IF (NGROUP .LE. 1)  GO TO 39
   40 SSY = 0.0D0
      YBAR = 0.0D0
      SUMY = 0.0D0
      SUMW = 0.0D0
      ISTOP=ISTART+NGROUP-1
      DO 38 J=ISTART,ISTOP
      SSY = SSY + SUMW/(SUMW+W(J)) * W(J) * (Y(J)-YBAR)**2
      YBAR = SUMW/(SUMW+W(J)) * YBAR   +  W(J)*Y(J)/(SUMW+W(J))
      SUMY = SUMY + W(J)*Y(J)
      SUMW = SUMW + W(J)
   38 CONTINUE
      NUMGRP = NUMGRP+1
      SUMSQ=SUMSQ+SSY
```

```
      SSY = SSY/(NGROUP-1)
      SUMN = SUMN+NGROUP-1
      BGSMSQ = BGSMSQ + SUMY**2/NGROUP
      SMGRPY = SMGRPY + SUMY
      WRITE(4,135) X(ISTART),NGROUP,SUMY,SSY
      GO TO 39
   42 NMIH = N-IH
      MNSQB = SUMRS/ABS(NMIH)
      B1 = SNGL(SUMRS) - SNGL(SUMSQ)
      DEGFB1 = NMIH-SUMN
      IF (DEGFB1 .NE. 0)  GO TO 45
      MNSQB1 = 0.0D0
      GO TO 48
   45 MNSQB1 = B1/DEGFB1
   48 IF (SUMN .EQ. 0)  GO TO 43
      BGSMSQ = BGSMSQ - SMGRPY**2/(SUMN+NUMGRP)
      NUMGRP = NUMGRP-1
      WRITE(4,136) SUMN,SUMSQ,NUMGRP,BGSMSQ
      MNSQB2 = SUMSQ/SUMN
      IF (DEGFB1 .EQ. 0)  GO TO 47
      F2 = B1*SUMN / (SUMSQ*DEGFB1)
      GO TO 44
   43 WRITE(4,138)
      MNSQB2 = 0.0D0
   47 F2 = 0.0D0
   44 C = SSB+SUMRS
      DEGFC = 1+NMIH
      MNSQC = C/DEGFC
      WRITE(4,137) SSW, SSB, F12B, NMIH, SUMRS, MNSQB, DEGFB1, B1,
```

```
     1       MNSQB1, F2, SUMN, SUMSQ, MNSQB2, DEGFC, C, MNSQC
        GO TO 1000
  999   STOP
  700 FORMAT(13A6,A2)
    5 FORMAT(3I5,E15.8)
    6 FORMAT(5E15.8)
   67 FORMAT(60H1REGRESSION ANALYSIS FOR TWO VARIABLES--(THROUGH THE ORI
     1GIN))
    7 FORMAT(59H1REGRESSION ANALYSIS FOR TWO VARIABLES--(THROUGH THE MEA
     1NS))
  800 FORMAT(1H0,13A6,A2)
   12 FORMAT(//10X,4HN = ,I5,10X,5HIH = ,I5,10X,5HIW = ,I5,10X,14HSTUDEN
     1T'S T = ,E15.8)
   23 FORMAT(//10X,37HINPUT VECTOR X (INDEPENDENT VARIABLE))
   50 FORMAT(/E17.8)
    8 FORMAT(//10X,35HINPUT VECTOR Y (DEPENDENT VARIABLE))
   24 FORMAT(//10X,24HINPUT VECTOR W (WEIGHTS))
   21 FORMAT(//10X,78HINPUT VECTOR P (POST EXPERIMENTAL EXPLORATORY POIN
     1TS OF X VARIABLE-ALWAYS TEN))
   25 FORMAT(//10X,48HPOST EXPERIMENTAL WEIGHTS Q ASSOCIATED WITH P(I))
   52 FORMAT(//10X,50HSTUDENT'S T FOR CALCULATION OF CONFIDENCE LEVELS =
     1,E14.8)
  102 FORMAT(////10X,15HSUM OF WX(I) = ,E15.8,5X,15HSUM OF WY(I) = ,E15.
     18,5X,14HSUM OF W(I) = ,E15.8)
  103 FORMAT(/10X,12HMEAN OF X = ,E15.8,24X,12HMEAN OF Y = ,E15.8)
  104 FORMAT(/10X,29HSS OF DEV. FROM MEAN OF WX = ,E15.8,/10X,29HSS OF D
     1EV. FROM MEAN OF WY = ,E15.8)
  105 FORMAT(/10X,24HSUM OF CROSS-PRODUCTS = ,E15.8)
  106 FORMAT(//10X,29HREGRESSION COEFFICIENT = B = ,E15.8,10X,32HSSB = S
```

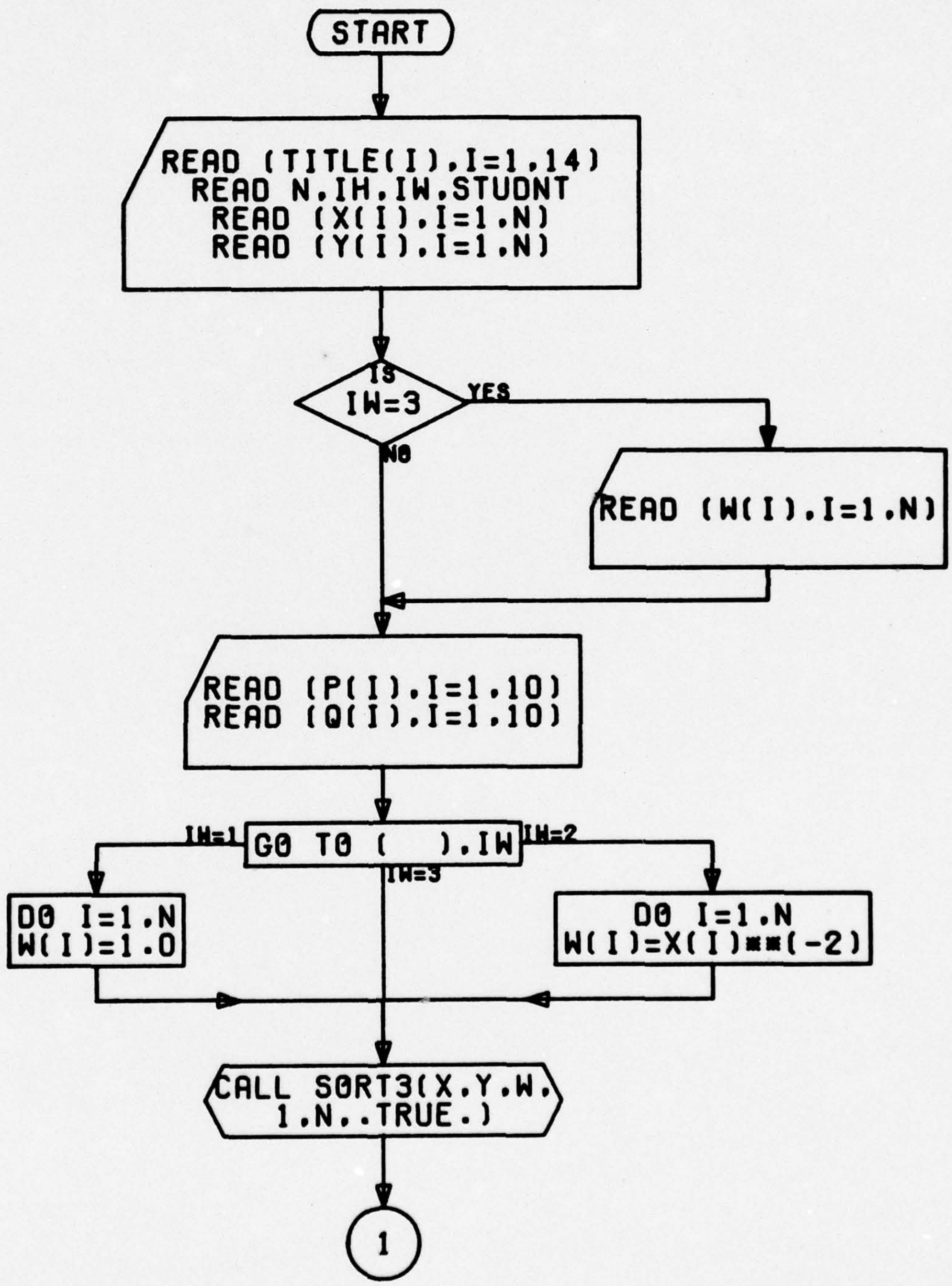
```
      1UM OF SQUARES DUE TO B = ,E15.8)
  107 FORMAT(//10X,16HINTERCEPT = A = ,E15.8)
  120 FORMAT(//10X,27HEQUATION = YHAT = A + BX = ,E15.8,5H  +  ,E15.8,2H
     1 X)
  108 FORMAT(//10X,18HPREDICTED Y = YHAT)
  109 FORMAT(//10X,33HPREDICTED EXPLORATORY Y = Y SUB E)
  110 FORMAT(//10X,45HDEVIATIONS OF Y FROM REGRESSION LINE (Y-YHAT))
  111 FORMAT(//10X,47HMULT. AND SIMPLE CORR. COEFF. SQUARED = R**2 = ,E1
     15.8)
  112 FORMAT(//10X,29HSTD. ERROR OF ESTIMATE = S = ,E15.8,10X,46HSSDEV =
     1 SUM SQUARES OF DEVIATIONS FROM LINE = ,E15.8)
  113 FORMAT(//10X,6HF(1,N-,I1,1H),1X,26HAS A TEST IF R = B = 0. = ,E15.
     15)
  114 FORMAT(//10X,28HSTD. ERROR OF B = S SUB B = ,E15.8)
  115 FORMAT(//10X,28HSTD. ERROR OF A = S SUB A = ,E15.8)
  116 FORMAT(//10X,6HF(1,N-,I1,1H),1X,82HAS A TEST IF A (INTERCEPT) = 0.
     1 OR THAT REGRESSION LINE GOES THROUGH THE ORIGIN = ,E15.8)
  117 FORMAT(//10X,76HSTD. ERROR OF EACH PREDICTED POINT (ON THE LINE),(
     1YHAT) AT EXPERIMENTAL X(I))
  118 FORMAT(//10X,107HSTD. ERROR OF EACH PREDICTED EXPLORATORY POINT (O
     1N THE LINE),(YHAT SUB E) AT POST EXPERIMENTAL X(I) OR P(I))
  119 FORMAT(//10X,61HSTD. ERROR OF EACH OBSERVED Y, (Y-YHAT) AT EXPERIM
     1ENTAL X(I),/10X,70HASSUMING THESE Y(I) ARE NEWLY OBSERVED INDIVIDU
     2ALS WITH WEIGHTS = W(I))
  125 FORMAT(//10X,104HSTD. ERROR OF EACH OBSERVED Y SUB E OR OF EACH (Y
     1 SUB E - YHAT SUB E) AT POST EXPERIMENTAL X(I) OR P(I),/10X,70HASS
     2UMING THESE Y(I) ARE NEWLY OBSERVED INDIVIDUALS WITH WEIGHTS = Q(I
     3))
  121 FORMAT(//10X,4HT(N-,I1,1H),1X,41HAS TEST OF EACH DEVIATION FROM RE
```

```
       1GRESSION)
    51 FORMAT(7F15.8)
   122 FORMAT(//10X,30HNORMALIZATION STATISTIC = U = ,E15.8)
   123 FORMAT(//10X,49HCONFIDENCE LIMITS FOR TRUE Y AT EXPERIMENTAL X(I))
   124 FORMAT(//10X,62HCONFIDENCE LIMITS FOR TRUE Y AT POST EXPERIMENTAL
       1X(I) OR P(I))
   126 FORMAT(//10X,49HCONFIDENCE LIMITS FOR TRUE Y AT EXPERIMENTAL X(I)/
       110X,70HASSUMING THESE Y(I) ARE NEWLY OBSERVED INDIVIDUALS WITH WEI
       2GHTS = W(I))
   127 FORMAT(//10X,62HCONFIDENCE LIMITS FOR TRUE Y AT POST EXPERIMENTAL
       1X(I) OR P(I)/10X,70HASSUMING THESE Y(I) ARE NEWLY OBSERVED INDIVID
       2UALS WITH WEIGHTS = Q(I))
    53 FORMAT(3(E17.8,2H ,,E17.8,2H ;))
   130 FORMAT(//10X,11HEFFECTIVE N//10X,4HNHAT)
   131 FORMAT(//10X,10HNHAT SUB E)
   132 FORMAT(//10X,1HN)
   133 FORMAT(//10X,7HN SUB E)
   134 FORMAT(///10X,64HVARY = VARIANCE OF Y WITHIN GROUPS AT ONE X, GROU
       1P SIZE, Y TOTAL)
   135 FORMAT(//T16,'AT X = ',E13.8,6X,'N =',I4,6X,'SUM Y = ',E13.8,6X,
       1     'VARIANCE = ',E13.8,' ;')
   136 FORMAT(//T11,'WITHIN GROUPS DEGREES OF FREEDOM AND SUM OF SQUARES'
       1     ,'  *  BETWEEN GROUPS DEGREES OF FREEDOM AND SUM OF SQUARES'/
       2     T64,'*'/28X,I5,14X,E13.8,T64,'*',20X,I5,14X,E13.8/T64,'*')
   137 FORMAT(1H1T26,'ANALYSIS OF VARIANCE'/1X,131('*')//8X,'SOURCE OF ',
       1     'VARIATION',11X,'DEGREES OF FREEDOM',9X,'SUM OF SQUARES',15X,
       2      'MEAN SQUARE',15X,'F'/1X,131('*')//T6,'SLOPE OF THE LINE',
       3     21X,'1',20X,E13.8,14X,E13.8,9X,E13.8//T6,'DEVIATIONS FROM ',
       4     'THE LINE',10X,I5,20X,E13.8,14X,E13.8//T12,'GROUP MEANS ',
```

```
     5      'FROM LINE',10X,I5,20X,E13.8,14X,E13.8,9X,E13.8//T12,
     6      'WITHIN GROUPS',18X,I5,20X,E13.8,14X,E13.8/1X,131('-')/T6,
     7      'TOTALS',28X,I5,20X,E13.8,14X,E13.8)
  138 FORMAT(//T11,'THERE ARE NO REPLICATES OF Y AT ANY SINGLE VALUE ',
     1      'OF X')
      END
```

APPENDIX D

S1281 PROGRAM REGRESS



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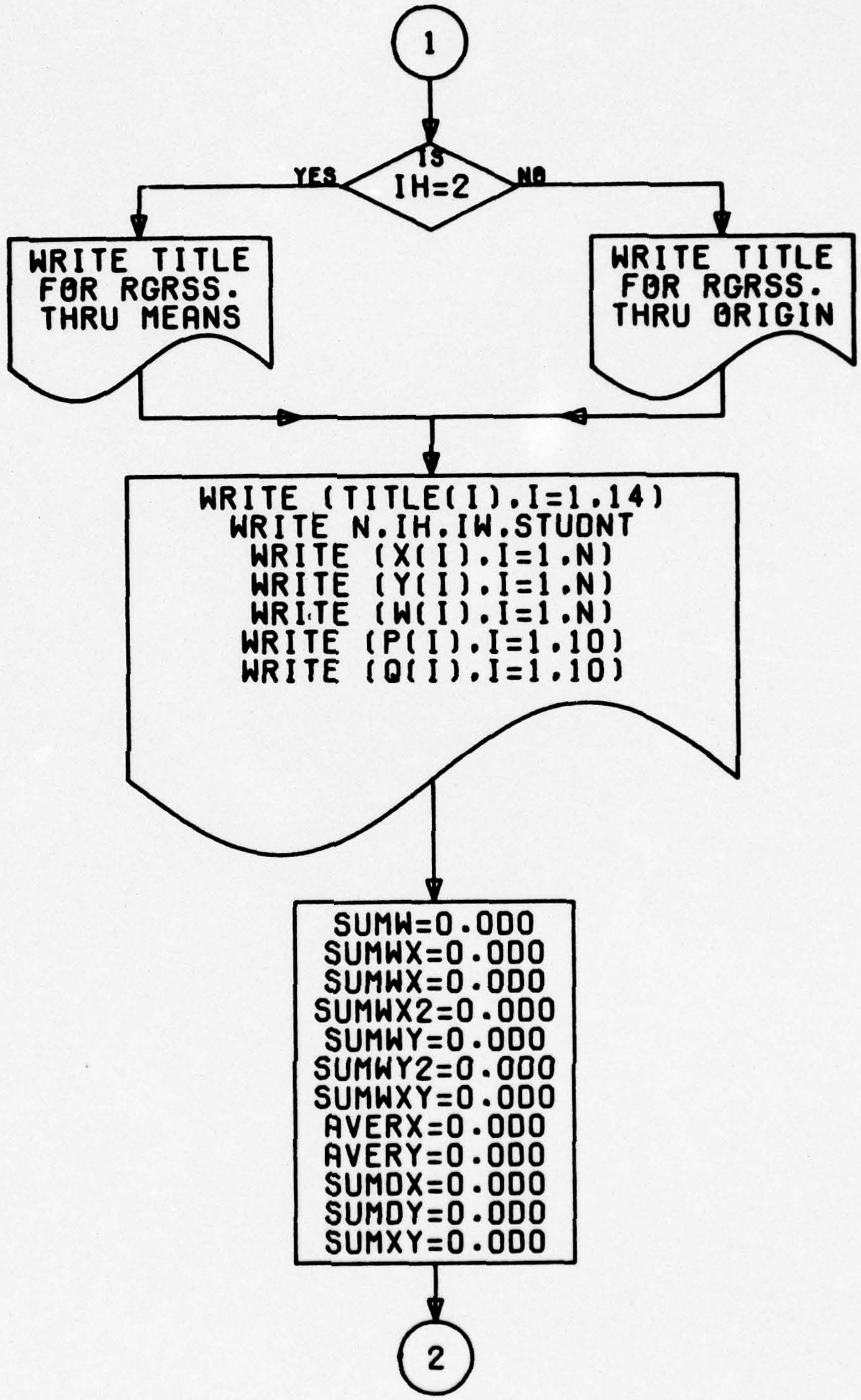
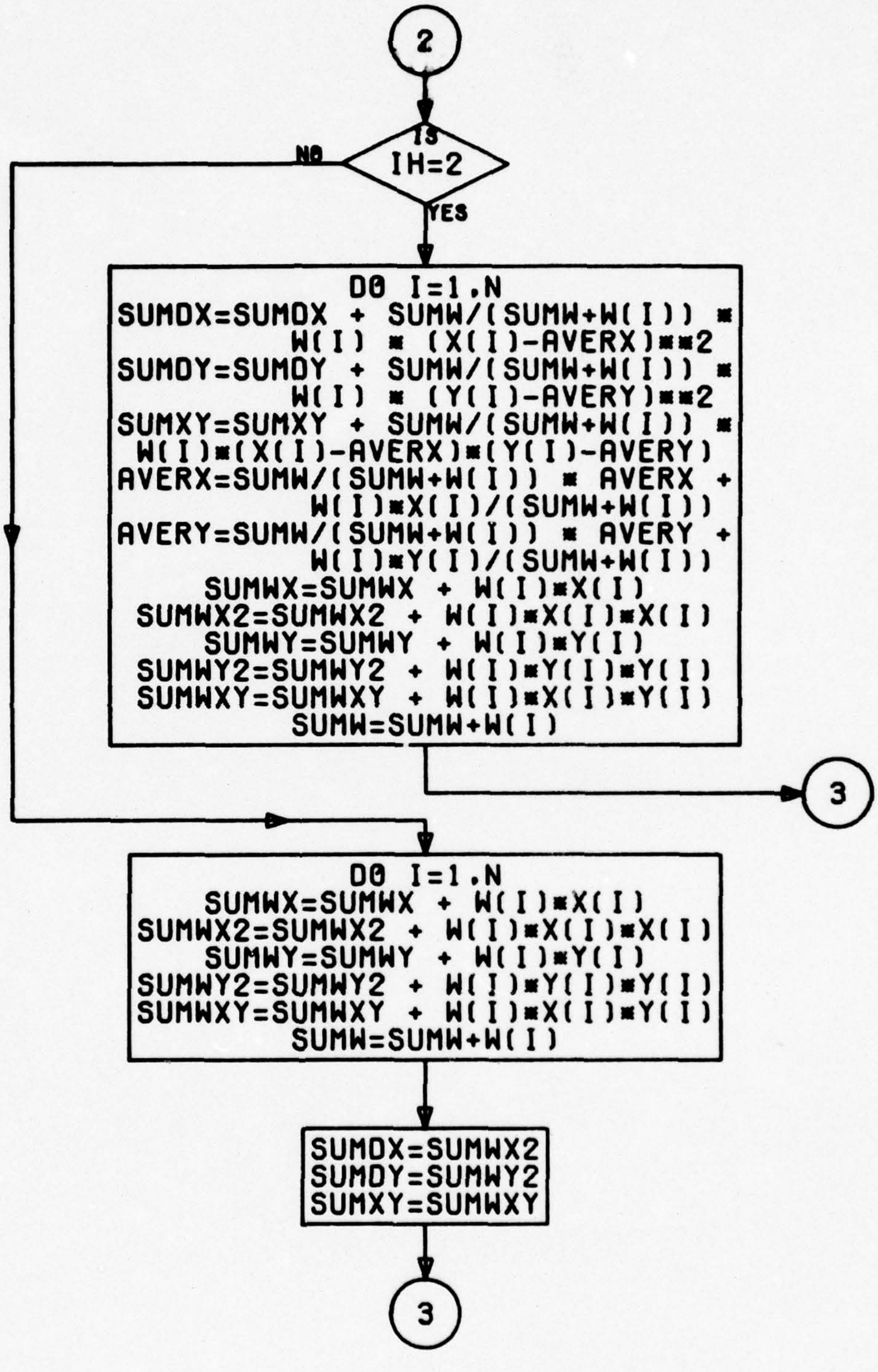
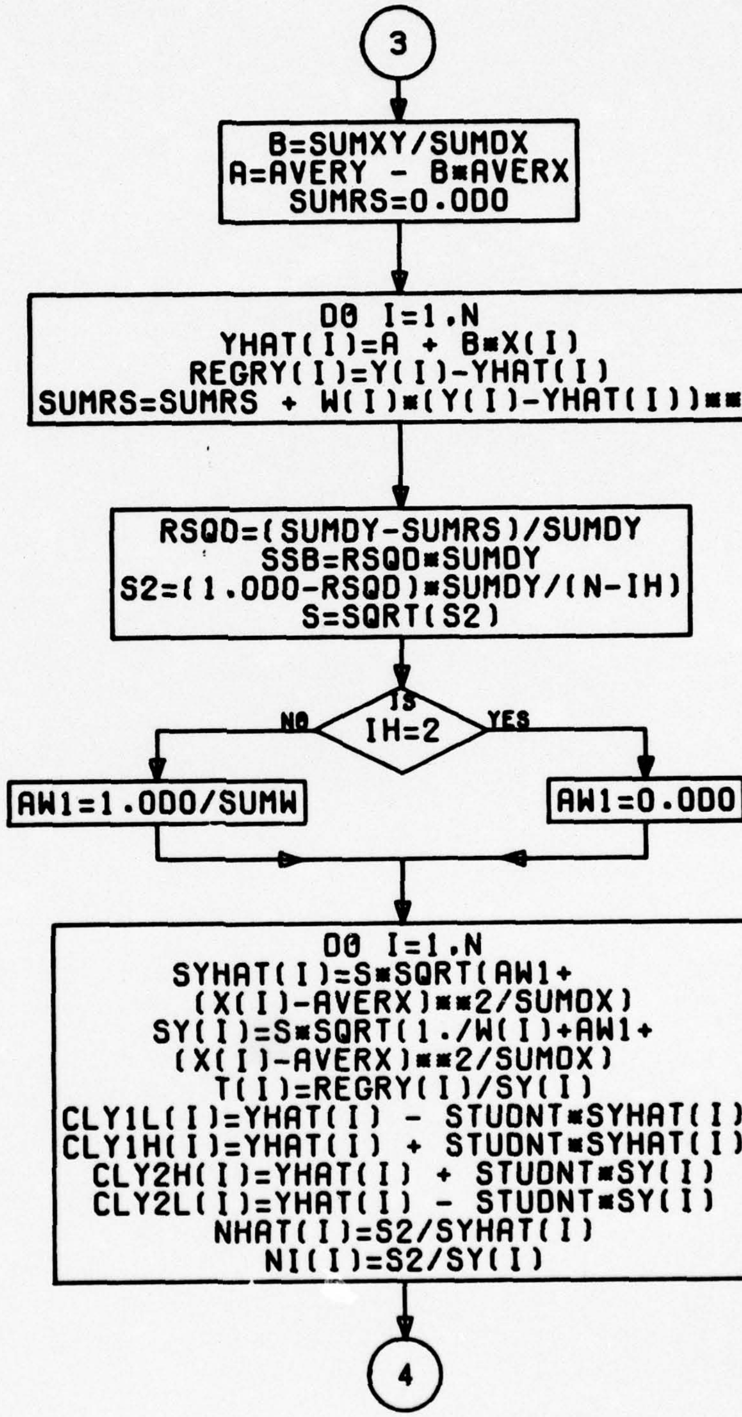
U. S. Navy Underwater Sound Laboratory Official Photograph
NP24 - 38752 - 4 - 70

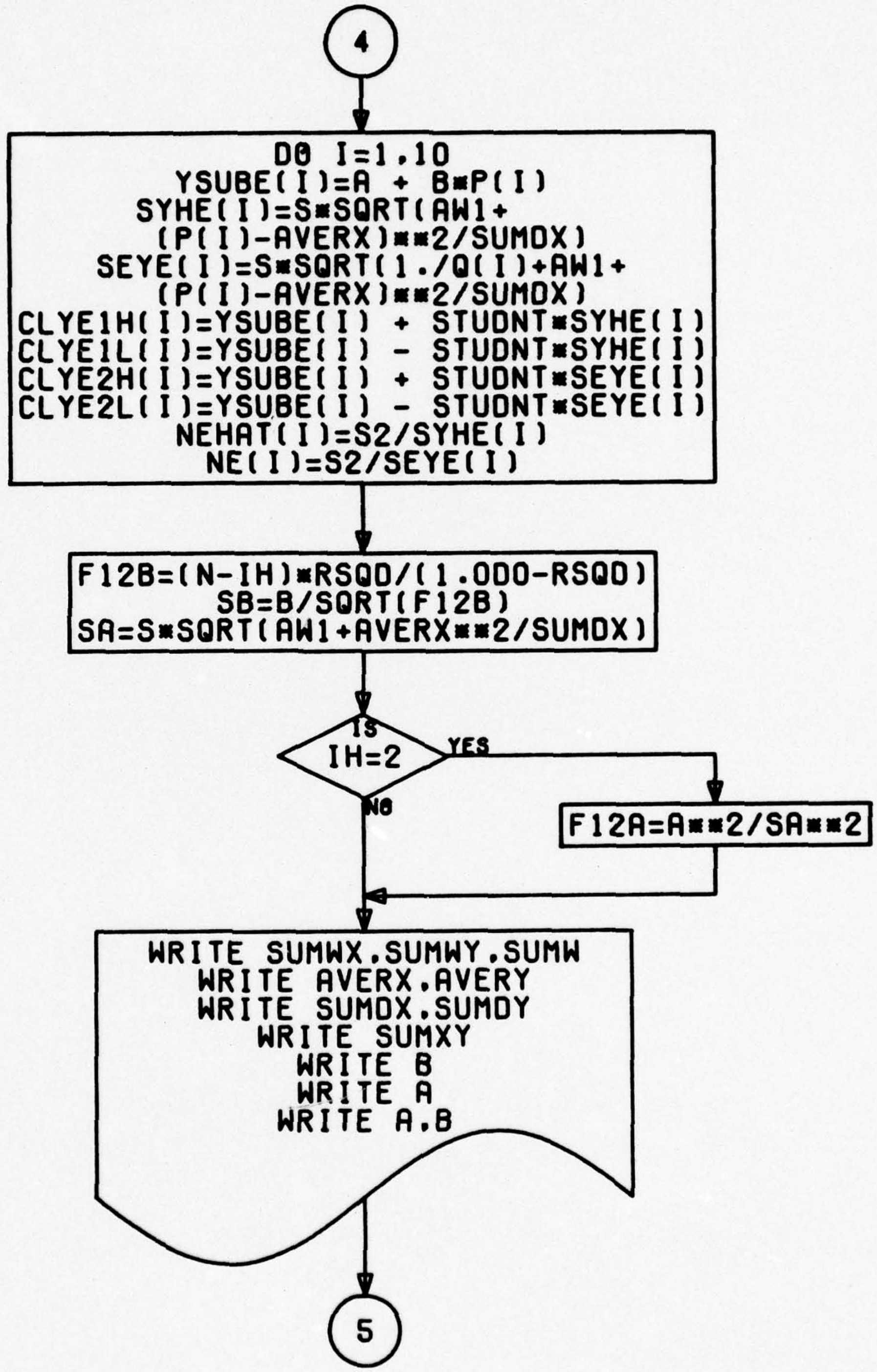


NUSL Tech Memo No. 2070-451-69

U. S. Navy Underwater Sound Laboratory Official Photograph
NP24 - 38750 - 4 - 70


2
IS
IH=2
NO
YES
DO I=1,N
SUMDX=SUMDX + SUMW/(SUMW+W(I)) *
W(I) * (X(I)-AVERX)**2
SUMDY=SUMDY + SUMW/(SUMW+W(I)) *
W(I) * (Y(I)-AVERY)**2
SUMXY=SUMXY + SUMW/(SUMW+W(I)) *
W(I)*(X(I)-AVERX)*(Y(I)-AVERY)
AVERX=SUMW/(SUMW+W(I)) * AVERX +
W(I)*X(I)/(SUMW+W(I))
AVERY=SUMW/(SUMW+W(I)) * AVERY +
W(I)*Y(I)/(SUMW+W(I))
SUMWX=SUMWX + W(I)*X(I)
SUMWX2=SUMWX2 + W(I)*X(I)*X(I)
SUMWY=SUMWY + W(I)*Y(I)
SUMWY2=SUMWY2 + W(I)*Y(I)*Y(I)
SUMWXY=SUMWXY + W(I)*X(I)*Y(I)
SUMW=SUMW+W(I)
3
DO I=1,N
SUMWX=SUMWX + W(I)*X(I)
SUMWX2=SUMWX2 + W(I)*X(I)*X(I)
SUMWY=SUMWY + W(I)*Y(I)
SUMWY2=SUMWY2 + W(I)*Y(I)*Y(I)
SUMWXY=SUMWXY + W(I)*X(I)*Y(I)
SUMW=SUMW+W(I)
SUMDX=SUMWX2
SUMDY=SUMWY2
SUMXY=SUMWXY
3


3
B=SUMXY/SUMDX
A=AVERY - B*AVERX
SUMRS=0.0D0
DO I=1,N
YHAT(I)=A + B*X(I)
REGRY(I)=Y(I)-YHAT(I)
SUMRS=SUMRS + W(I)*(Y(I)-YHAT(I))**2
RSQD=(SUMDY-SUMRS)/SUMDY
SSB=RSQD*SUMDY
S2=(1.0D0-RSQD)*SUMDY/(N-IH)
S=SQRT(S2)
IS
IH=2
NO
YES
AW1=1.0D0/SUMW
AW1=0.0D0
DO I=1,N
SYHAT(I)=S*SQRT(AW1+
(X(I)-AVERX)**2/SUMDX)
SY(I)=S*SQRT(1./W(I)+AW1+
(X(I)-AVERX)**2/SUMDX)
T(I)=REGRY(I)/SY(I)
CLY1L(I)=YHAT(I) - STUDNT*SYHAT(I)
CLY1H(I)=YHAT(I) + STUDNT*SYHAT(I)
CLY2H(I)=YHAT(I) + STUDNT*SY(I)
CLY2L(I)=YHAT(I) - STUDNT*SY(I)
NHAT(I)=S2/SYHAT(I)
NI(I)=S2/SY(I)
4



NUSL Tech Memo No. 2070-451-69

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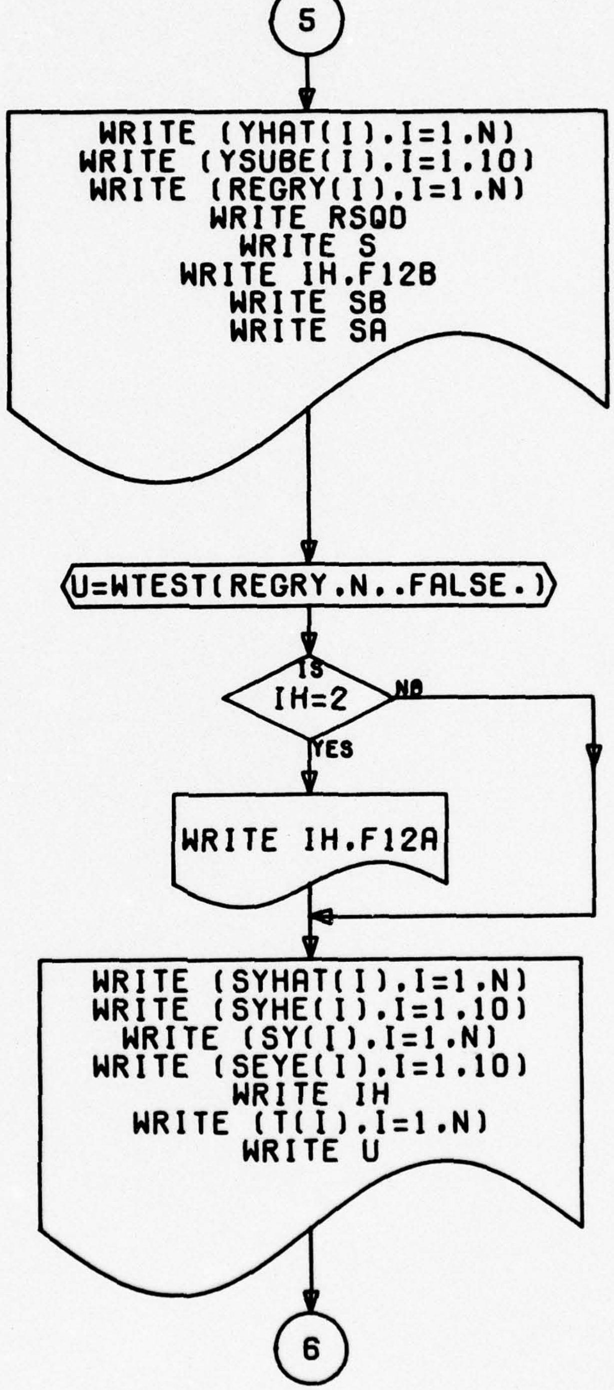
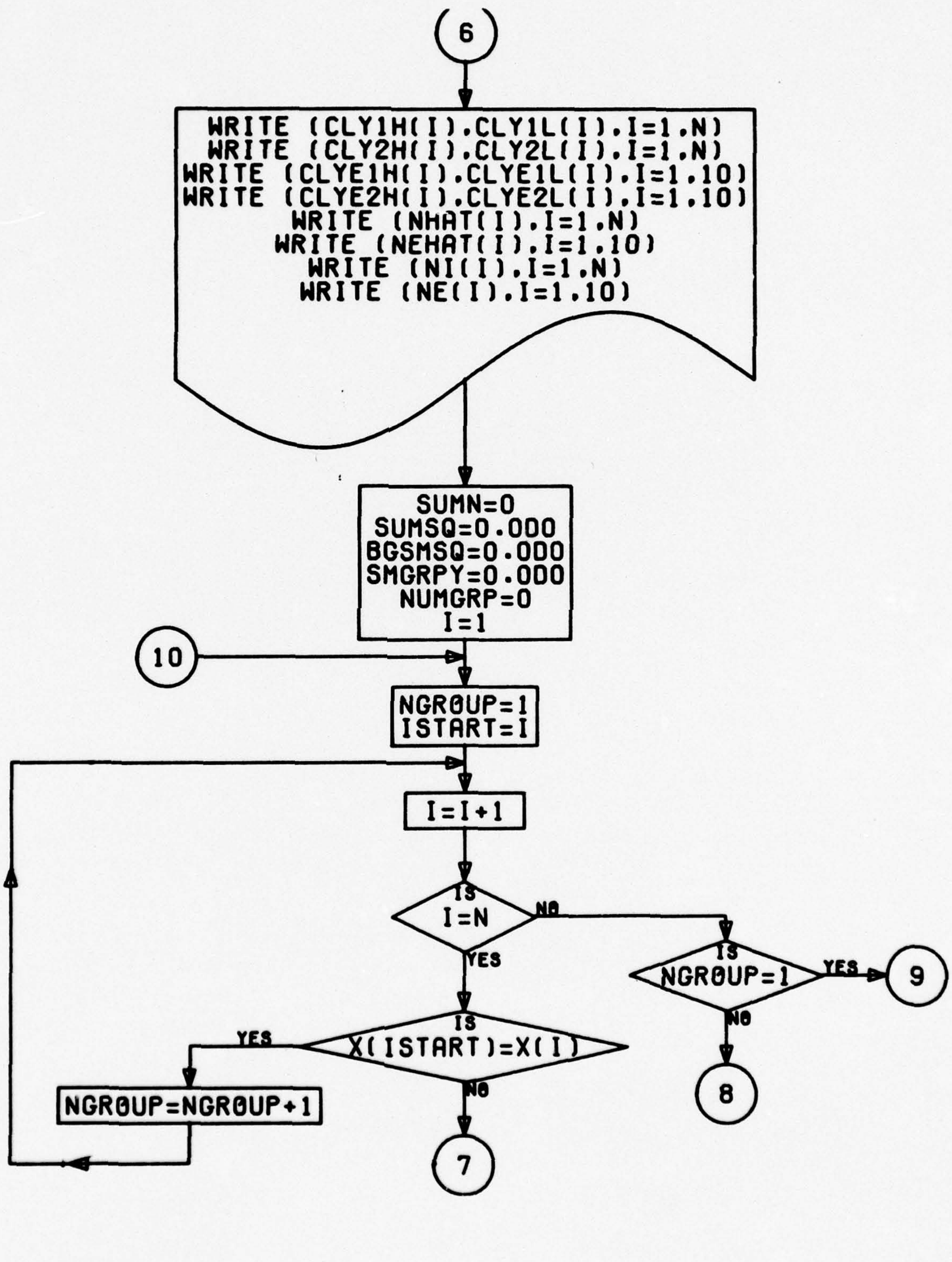
NP24 - 38755 - 4 - 70



NUSL Tech Memo No. 2070-451-69

U. S. Navy Underwater Sound Laboratory
NP24 - 38758 - 4 - 70

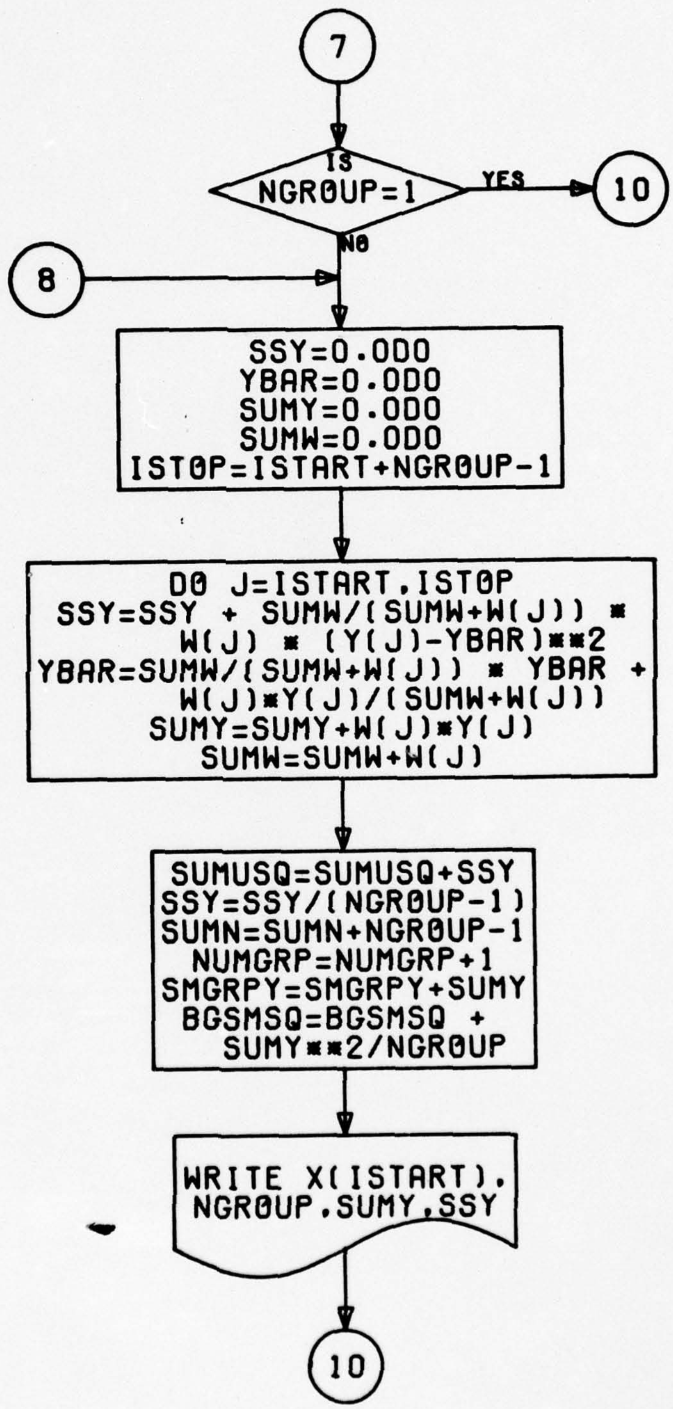
Official Photograph
Confidential

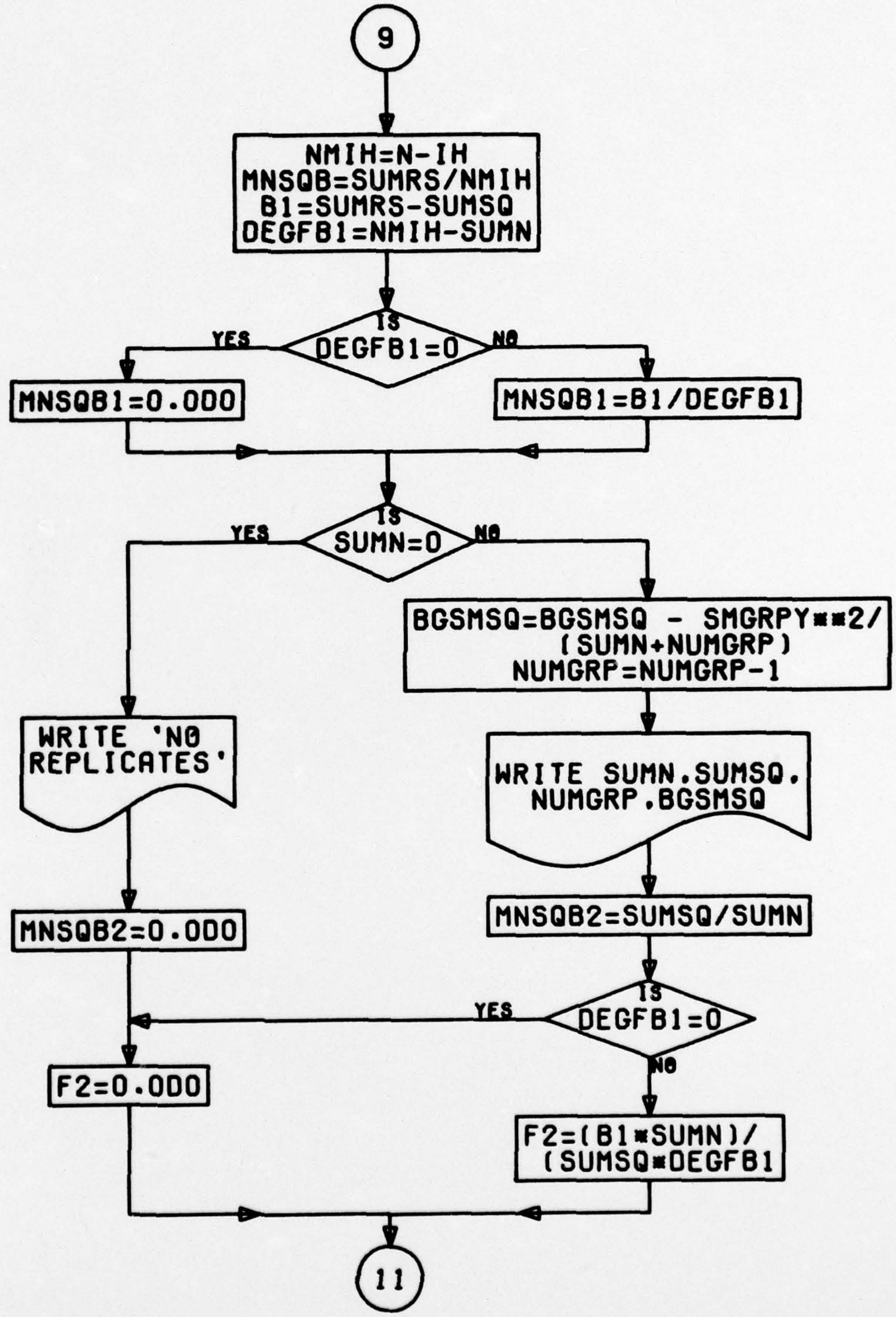


NUSL Tech Memo No. 2070-451-69

U. S. Navy Underwater Sound Laboratory
NP24 - 38751 - 4 - 70

Official Photograph

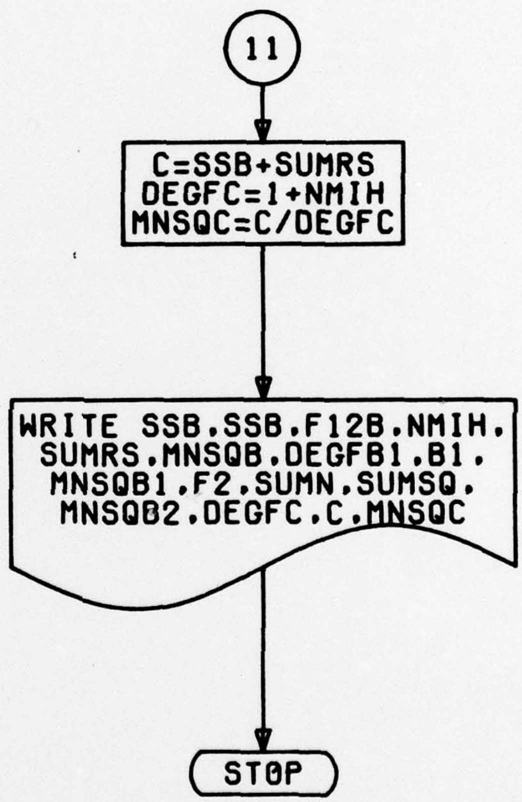

7
IS
NGROUP=1
YES
10
NO
8
SSY=0.0D0
YBAR=0.0D0
SUMY=0.0D0
SUMW=0.0D0
ISTOP=ISTART+NGROUP-1
DO J=ISTART,ISTOP
SSY=SSY + SUMW/(SUMW+W(J)) *
W(J) * (Y(J)-YBAR)**2
YBAR=SUMW/(SUMW+W(J)) * YBAR +
W(J)*Y(J)/(SUMW+W(J))
SUMY=SUMY+W(J)*Y(J)
SUMW=SUMW+W(J)
SUMUSQ=SUMUSQ+SSY
SSY=SSY/(NGROUP-1)
SUMN=SUMN+NGROUP-1
NUMGRP=NUMGRP+1
SMGRPY=SMGRPY+SUMY
BGSMSQ=BGSMSQ +
SUMY**2/NGROUP
WRITE X(ISTART),
NGROUP,SUMY,SSY
10



NUSL Tech Memo No. 2070-451-69

U. S. Navy Underwater Sound Laboratory
NP24 - 38753 - 4 - 70

Official Photograph



NUSL Tech Memo No. 2070-451-69

U. S. Navy Underwater Sound Laboratory
NP24 - 38759 - 4 - 70

Official Photograph

```
REGRESSION ANALYSIS FOR TWO VARIABLES--(THROUGH THE MEANS)

EXAMPLE FOR PROGRAM REGRESS FROM BROWNLEE   PAGE 324   NO GROUP VARIANCE

      N =      5           IN =      2           IW =      3          STUDENT'S T =     .45410000+01

      INPUT VECTOR X (INDEPENDENT VARIABLE)
 .26500000+02    .30500000+02    .40500000+02    .48800000+02    .57800000+02

      INPUT VECTOR Y (DEPENDENT VARIABLE)
 .37650000+01    .55100000+01    .85500000+01    .10630000+02    .11940000+02

      INPUT VECTOR W (WEIGHTS)
 .20000000+01    .20000000+01    .10000000+01    .10000000+01    .10000000+01

      INPUT VECTOR P (POST EXPERIMENTAL EXPLORATORY POINTS OF X VARIABLE-ALWAYS TEN)
 .10000000+02    .15000000+02    .50000000+02    .60000000+02    .65000000+02    .70000000+02    .75000000+02
 .60000000+02    .80000000+02    .80000000+02

      POST EXPERIMENTAL WEIGHTS Q ASSOCIATED WITH P(I)
 .10000000+02    .10000000+02    .10000000+02    .10000000+02    .10000000+02    .20000000+02    .30000000+02
 .40000000+02    .10000000+03    .10000000+04


      SUM OF WX(I) =    .24910000+03     SUM OF WY(I) =    .49710000+02     SUM OF W(I) =    .70000000+01

      MEAN OF X =    .35585714+02                       MEAN OF Y =    .71014286+01

      SS OF DEV. FROM MEAN OF WX =    .11991286+04
      SS OF DEV, FROM MEAN OF WY =    .65023636+02

      SUM OF CROSS-PRODUCTS =    .27748015+03

      REGRESSION COEFFICIENT = B =    .23140149+00          SSB = SUM OF SQUARES DUE TO B =    .64209320+02

      INTERCEPT = A =   -.11331589+01

      EQUATION = YHAT = A + BX =   -.11331589+01  +    .23140149+00 X

      PREDICTED Y = YHAT
 .30105717+01    .39245667+01    .82386016+01    .10159234+02    .12241847+02

      PREDICTED EXPLORATORY Y = Y SUB E
 .11808560+01    .23378635+01    .10436916+02    .12750931+02    .13907938+02    .15064946+02    .16221953+02
 .17378960+02    .17378960+02    .17378960+02
```

```
DEVIATIONS OF Y FROM REGRESSION LINE (Y-YHAT)
.17442826+00     -.41456666+00     .31139839+00     .47076595+00     -.30184734+00

MULT. AND SIMPLE CORR. COEFF. SQUARED = R**2 =     .98747662+00

STD. ERROR OF ESTIMATE = S =     .52099776+00          SSDEV = SUM SQUARES OF DEVIATIONS FROM LINE =     .81431600+00

F(1,N-2) AS A TEST IF R = B = 0. =     .23655185+03

STD. ERROR OF B = S SUB B =     .15045374-01

STD. ERROR OF A = S SUB A =     .57046517+00

F(1,N-2) AS A TEST IF A (INTERCEPT) = 0. OR THAT REGRESSION LINE GOES THROUGH THE ORIGIN =     .39456935+01

STD. ERROR OF EACH PREDICTED POINT (ON THE LINE),(YHAT) AT EXPERIMENTAL X(I)
.30046699+00     .21126221+00     .21034160+00     .27982835+00     .38791939+00

STD. ERROR OF EACH PREDICTED EXPLORATORY POINT (ON THE LINE),(YHAT SUB E) AT POST EXPERIMENTAL X(I) OR P(I)
.45230971+00     .36701946+00     .29293142+00     .41677625+00     .48430260+00     .55395732+00     .62484326+00
.69664026+00     .69664026+00     .69664026+00

STD. ERROR OF EACH OBSERVED Y, (Y-YHAT) AT EXPERIMENTAL X(I),
ASSUMING THESE Y(I) ARE NEWLY OBSERVED INDIVIDUALS WITH WEIGHTS = W(I)
.47540694+00     .42467759+00     .56185615+00     .59139037+00     .64955376+00

STD. ERROR OF EACH OBSERVED Y SUB E OR OF EACH (Y SUB E - YHAT SUB E) AT POST EXPERIMENTAL X(I) OR P(I),
ASSUMING THESE Y(I) ARE NEWLY OBSERVED INDIVIDUALS WITH WEIGHTS = Q(I)
.46271452+00     .40230231+00     .33608434+00     .44815880+00     .51163499+00     .56607477+00     .63204198+00
.70149384+00     .69658574+00     .69683565+00

T(N-2) AS TEST OF EACH DEVIATION FROM REGRESSION
 .367            -.976             .554             .796             -.465

NORMALIZATION STATISTIC = J =     .90305057+00

CONFIDENCE LIMITS FOR TRUE Y AT EXPERIMENTAL X(I)
.49750631+01 ,    .22460603+01 ;    .68839284+01 ,    .49652449+01 ;    .91937636+01 ,    .72834395+01 ;
.11429935+02 ,    .88885336+01 ;    .14003389+02 ,    .10480306+02 ;

CONFIDENCE LIMITS FOR TRUE Y AT POST EXPERIMENTAL X(I) OR P(I)
.31443377+01 ,    -.78262562+00 ;    .40044988+01 ,    .67122814+00 ;    .11767117+02 ,    .91067142+01 ;
```

```
.14643512+02 ,      .10856350+02 ;      .16107519+02 ,      .11708357+02 ;      .17580466+02 ,      .12549425+02 ;
.19059366+02 ,      .13364540+02 ;      .20542404+02 ,      .14215517+02 ;      .20542404+02 ,      .14215517+02 ;
.20542404+02 ,      .14215517+02 ;

     CONFIDENCE LIMITS FOR TRUE Y AT EXPERIMENTAL X(I)
     ASSUMING THESE Y(I) ARE NEWLY OBSERVED INDIVIDUALS WITH WEIGHTS = W(I)
.57693946+01 ,      .14517488+01 ;      .78530475+01 ,      .39961258+01 ;      .10789990+02 ,      .56872128+01 ;
.12844736+02 ,      .74737504+01 ;      .15191471+02 ,      .92922237+01 ;

     CONFIDENCE LIMITS FOR TRUE Y AT POST EXPERIMENTAL X(I) OR P(I)
     ASSUMING THESE Y(I) ARE NEWLY OBSERVED INDIVIDUALS WITH WEIGHTS = Q(I)
.32820427+01 ,     -.92033058+00 ;      .41647183+01 ,      .51100871+00 ;      .11963075+02 ,      .89107568+01 ;
.14786020+02 ,      .10715642+02 ;      .16231272+02 ,      .11584604+02 ;      .17635491+02 ,      .12494400+02 ;
.19092055+02 ,      .13351651+02 ;      .20564444+02 ,      .14193477+02 ;      .20551238+02 ,      .14206683+02 ;
.20543268+02 ,      .14214633+02 ;

     EFFECTIVE N

     NHAT
.30062174+01     .60617436+01     .61350846+01     .34664767+01     .18036015+01

     NHAT SUB E
.14318475+01     .20150657+01     .31632957+01     .15626647+01     .11568969+01     .88454331+00     .69523165+00
.55931256+00     .55931256+00     .55931256+00

     N
.12009936+01     .15053573+01     .85984750+00     .77610988+00     .64334137+00

     N SUB E
.12677845+01     .18771297+01     .24031183+01     .13514745+01     .10369343+01     .84707938+00     .67948499+00
.55139965+00     .55629166+00     .55899991+00

     VART = VARIANCE OF Y WITHIN GROUPS AT ONE X, GROUP SIZE, Y TOTAL

     THERE ARE NO REPLICATES OF Y AT ANY SINGLE VALUE OF X
```

ANALYSIS OF VARIANCE

SOURCE OF VARIATION	DEGREES OF FREEDOM	SUM OF SQUARES	MEAN SQUARE	F
SLOPE OF THE LINE	1	.64209320+02	.64209320+02	.23655185+03
DEVIATIONS FROM THE LINE	3	.81431600+00	.27143867+00	
GROUP MEANS FROM LINE	3	.81431600+00	.27143867+00	.00000000
WITHIN GROUPS	0	.00000000	.00000000	
TOTALS	4	.65023636+02	.16255909+02	

```
REGRESSION ANALYSIS FOR TWO VARIABLES--(THROUGH THE MEANS)

EXAMPLE FOR PROGRAM REGRESS FROM BROWNLEE  PAGE 324

     N =      7           IH =      2           IW =      1          STUDENT'S T =    .33650000+01

     INPUT VECTOR X (INDEPENDENT VARIABLE)
 .20500000+02    .20500000+02    .30500000+02    .30500000+02    .40500000+02    .48800000+02    .57800000+02

     INPUT VECTOR Y (DEPENDENT VARIABLE)
 .39200000+01    .36500000+01    .58200000+01    .52000000+01    .85500000+01    .10630000+02    .11940000+02

     INPUT VECTOR W (WEIGHTS)
 .10000000+01    .10000000+01    .10000000+01    .10000000+01    .10000000+01    .10000000+01    .10000000+01

     INPUT VECTOR P (POST EXPERIMENTAL EXPLORATORY POINTS OF X VARIABLE-ALWAYS TEN)
 .10000000+02    .15000000+02    .50000000+02    .60000000+02    .65000000+02    .70000000+02    .75000000+02
 .80000000+02    .80000000+02    .80000000+02

     POST EXPERIMENTAL WEIGHTS Q ASSOCIATED WITH P(I)
 .10000000+02    .10000000+02    .10000000+02    .10000000+02    .10000000+02    .20000000+02    .30000000+02
 .40000000+02    .10000000+03    .10000000+04


     SUM OF WX(I) =    .24910000+03     SUM OF WY(I) =    .49710000+02     SUM OF W(I) =    .70000000+01

     MEAN OF X =    .35585714+02                     MEAN OF Y =    .71014286+01

     SS OF DEV. FROM MEAN OF WX =    .11991286+04
     SS OF DEV. FROM MEAN OF WY =    .65252286+02

     SUM OF CROSS-PRODUCTS =    .27748015+03

     REGRESSION COEFFICIENT = B =    .23140149+00          SSB = SUM OF SQUARES DUE TO B =    .64209320+02

     INTERCEPT = A =   -.11331589+01

     EQUATION = YHAT = A + BX =   -.11331589+01  +    .23140149+00 X

     PREDICTED Y = YHAT
 .36105717+01    .36105717+01    .59245867+01    .59245867+01    .82386016+01    .10159234+02    .12241847+02

     PREDICTED EXPLORATORY Y = Y SUB E
 .11808560+01    .23378635+01    .10436916+02    .12750931+02    .13907938+02    .15064946+02    .16221953+02
 .17378960+02    .17378960+02    .17378960+02
```

```
     DEVIATIONS OF Y FROM REGRESSION LINE (Y-YHAT)
.30942627+00     .39428264-01     -.10458666+00     -.72458667+00     .31139839+00     .47076595+00     -.30184734+00

     MULT. AND SIMPLE CORR. COEFF. SQUARED = R**2 =    .98401641+00

     STD. ERROR OF ESTIMATE = S =    .45672005+00          SSDEV = SUM SQUARES OF DEVIATIONS FROM LINE =    .10429660+01

     F(1,N-2) AS A TEST IF R = B = 0. =    .30782077+03

     STD. ERROR OF B = S SUB B =    .13189162-01

     STD. ERROR OF A = S SUB A =    .50008445+00

     F(1,N-2) AS A TEST IF A (INTERCEPT) = 0. OR THAT REGRESSION LINE GOES THROUGH THE ORIGIN =    .51344615+01

     STD. ERROR OF EACH PREDICTED POINT (ON THE LINE),(YHAT) AT EXPERIMENTAL X(I)
.26341462+00     .26341462+00     .18519769+00     .18519769+00     .18439103+00     .24530473+00     .34006012+00

     STD. ERROR OF EACH PREDICTED EXPLORATORY POINT (ON THE LINE),(YHAT SUB E) AT POST EXPERIMENTAL X(I) OR P(I)
.37904395+00     .32173671+00     .25679122+00     .36535678+00     .42462225+00     .48561325+00     .54775368+00
.61069278+00     .61069278+00     .61069278+00

     STD. ERROR OF EACH OBSERVED Y, (Y-YHAT) AT EXPERIMENTAL X(I),
     ASSUMING THESE Y(I) ARE NEWLY OBSERVED INDIVIDUALS WITH WEIGHTS = W(I)
.52723653+00     .52723653+00     .49284020+00     .49284020+00     .49253756+00     .51842802+00     .56941556+00

     STD. ERROR OF EACH OBSERVED Y SUB E OR OF EACH (Y SUB E - YHAT SUB E) AT POST EXPERIMENTAL X(I) OR P(I),
     ASSUMING THESE Y(I) ARE NEWLY OBSERVED INDIVIDUALS WITH WEIGHTS = Q(I)
.40562746+00     .35266656+00     .29462018+00     .39286754+00     .44851240+00     .49623572+00     .55406426+00
.61494756+00     .61259824+00     .61086354+00

     T(N-2) AS TEST OF EACH DEVIATION FROM REGRESSION
  .587           .075           -.212           -1.470           .632           .908           -.530

     NORMALIZATION STATISTIC = U =    .93771549+00

     CONFIDENCE LIMITS FOR TRUE Y AT EXPERIMENTAL X(I)
.44969619+01 ,     .27241816+01 ;     .44969619+01 ,     .27241816+01 ;     .65477775+01 ,     .53013958+01 ;
.65477775+01 ,     .53013958+01 ;     .88590773+01 ,     .76181258+01 ;     .10984684+02 ,     .93337836+01 ;
.13386150+02 ,     .11097545+02 ;

     CONFIDENCE LIMITS FOR TRUE Y AT POST EXPERIMENTAL X(I) OR P(I)
```

```
.24563389+01 ,    -.94626614-01 ;     .34205142+01 ,     .12552128+01 ;     .11301018+02 ,     .95728134+01 ;
.13980356+02 ,     .11521505+02 ;     .15336792+02 ,     .12479084+02 ;     .16699034+02 ,     .13430857+02 ;
.18065144+02 ,     .14376762+02 ;     .19433941+02 ,     .15323979+02 ;     .19433941+02 ,     .15323979+02 ;
.19433941+02 ,     .15323979+02 ;

     CONFIDENCE LIMITS FOR TRUE Y AT EXPERIMENTAL X(I)
     ASSUMING THESE Y(I) ARE NEWLY OBSERVED INDIVIDUALS WITH WEIGHTS = W(I)
.53847293+01 ,     .18364141+01 ;     .53847293+01 ,     .18364141+01 ;     .75829939+01 ,     .42661794+01 ;
.75829939+01 ,     .42661794+01 ;     .98959904+01 ,     .65812128+01 ;     .11903744+02 ,     .84147239+01 ;
.14157931+02 ,     .10325764+02 ;

     CONFIDENCE LIMITS FOR TRUE Y AT POST EXPERIMENTAL X(I) OR P(I)
     ASSUMING THESE Y(I) ARE NEWLY OBSERVED INDIVIDUALS WITH WEIGHTS = Q(I)
.25457924+01 ,    -.18408032+00 ;     .35245932+01 ,     .11511338+01 ;     .11428313+02 ,     .94455189+01 ;
.14072930+02 ,     .11428931+02 ;     .15417182+02 ,     .12398694+02 ;     .16734779+02 ,     .13395112+02 ;
.18086379+02 ,     .14357527+02 ;     .19448259+02 ,     .15309662+02 ;     .19439680+02 ,     .15318240+02 ;
.19434516+02 ,     .15323405+02 ;

     EFFECTIVE N

     NHAT
.30062174+01    .30062174+01    .60817429+01    .60817429+01    .61350846+01    .34664767+01    .18038015+01

     NHAT SUB E
.14518475+01    .20150857+01    .31632957+01    .15626647+01    .11568969+01    .88454330+00    .69523164+00
.55931257+00    .55931257+00    .55931257+00

     N
.75036798+00    .75036798+00    .85879182+00    .85879182+00    .85984750+00    .77610988+00    .64334137+00

     N SUB E
.12677646+01    .16771297+01    .24031183+01    .13514745+01    .10369343+01    .84707937+00    .67948500+00
.55159965+00    .55620166+00    .55899991+00

     VART = VARIANCE OF Y WITHIN GROUPS AT ONE X, GROUP SIZE, Y TOTAL

          AT X =  .20000000+02     N =  2     SUM Y =  .75700000+01     VARIANCE =  .36450003-01 ;

          AT X =  .30000000+02     N =  2     SUM Y =  .11020000+02     VARIANCE =  .19220000+00 ;

     WITHIN GROUPS DEGREES OF FREEDOM AND SUM OF SQUARES  *  BETWEEN GROUPS DEGREES OF FREEDOM AND SUM OF SQUARES
                                                          *
                    2                  .22865001+00       *                    1                  .29756249+01
                                                          *
```

ANALYSIS OF VARIANCE

SOURCE OF VARIATION	DEGREES OF FREEDOM	SUM OF SQUARES	MEAN SQUARE	F
SLOPE OF THE LINE	1	.64209320+02	.64209320+02	.30782077+03
DEVIATIONS FROM THE LINE	5	.10429660+01	.20859320+00	
GROUP MEANS FROM LINE	3	.81431599+00	.27143866+00	.23742721+01
WITHIN GROUPS	2	.22865001+00	.11432500+00	
TOTALS	6	.65252286+02	.10875381+02	

```
REGRESSION ANALYSIS FOR TWO VARIABLES--(THROUGH THE ORIGIN)

NEW TEST CASE FOR WEIGHTED REGRESSION WITH GROUPS

      N =      6            IM =      1            IW =      3           STUDENT'S T =     .20000000+01

      INPUT VECTOR X (INDEPENDENT VARIABLE)
 .10000000+02    .10000000+02    .20000000+02    .20000000+02    .20000000+02    .40000000+02

      INPUT VECTOR Y (DEPENDENT VARIABLE)
 .20000000+01    .50000000+01    .70000000+01    .60000000+01    .40000000+01    .80000000+01

      INPUT VECTOR W (WEIGHTS)
 .10000000+01    .20000000+01    .10000000+01    .30000000+01    .20000000+01    .10000000+01

      INPUT VECTOR P (POST EXPERIMENTAL EXPLORATORY POINTS OF X VARIABLE-ALWAYS TEN)
 .50000000+02    .55000000+02    .60000000+02    .65000000+02    .70000000+02    .75000000+02    .80000000+02
 .85000000+02    .90000000+02    .95000000+02

      POST EXPERIMENTAL WEIGHTS G ASSOCIATED WITH P(I)
 .10000000+02    .15000000+02    .20000000+02    .25000000+02    .30000000+02    .35000000+02    .40000000+02
 .45000000+02    .50000000+02    .55000000+02


      SUM OF WX(I) =    .19000000+03     SUM OF WY(I) =    .53000000+02     SUM OF W(I) =    .10000000+02

      MEAN OF X =     .00000000                           MEAN OF Y =     .00000000

      SS OF DEV. FROM MEAN OF WX =     .43000000+04
      SS OF DEV. FROM MEAN OF WY =     .30700000+03

      SUM OF CROSS-PRODUCTS =     .11000000+04

      REGRESSION COEFFICIENT = B =     .25581395+00          SSB = SUM OF SQUARES DUE TO B =     .28139535+03

      INTERCEPT = A =     .00000000

      EQUATION = YHAT = A + BX =     .00000000     +     .25581395+00 X

      PREDICTED Y = YHAT
 .25581395+01    .25581395+01    .51162791+01    .51162791+01    .51162791+01    .10232558+02

      PREDICTED EXPLORATORY Y = Y SUB E
 .12790698+02    .14069767+02    .15348837+02    .16627907+02    .17906977+02    .19186046+02    .20465116+02
 .21744186+02    .23023256+02    .24302325+02
```

```
     DEVIATIONS OF Y FROM REGRESSION LINE (Y-YHAT)
-.55813953+00    .24418605+01    .18837209+01    .88372093+00    -.11162791+01    -.22325581+01

     MULT. AND SIMPLE CORR. COEFF. SQUARED = R**2 =    .91659723+00

     STD. ERROR OF ESTIMATE = S =    .22629472+01         SSDEV = SUM SQUARES OF DEVIATIONS FROM LINE =    .25604651+02

     F(1,N-1) AS A TEST IF R = B = 0. =    .54950045+02

     STD. ERROR OF B = S SUB B =    .34509622-01

     STD. ERROR OF A = S SUB A =    .00000000

     STD. ERROR OF EACH PREDICTED POINT (ON THE LINE),(YHAT) AT EXPERIMENTAL X(I)
 .34509622+00    .34509622+00    .69019244+00    .69019244+00    .69019244+00    .13803849+01

     STD. ERROR OF EACH PREDICTED EXPLORATORY POINT (ON THE LINE),(YHAT SUB E) AT POST EXPERIMENTAL X(I) OR P(I)
 .17254611+01    .18980292+01    .20705773+01    .22431254+01    .24156735+01    .25882216+01    .27607698+01
 .29333178+01    .31058660+01    .32784141+01

     STD. ERROR OF EACH OBSERVED Y, (Y-YHAT) AT EXPERIMENTAL X(I),
     ASSUMING THESE Y(I) ARE NEWLY OBSERVED INDIVIDUALS WITH WEIGHTS = W(I)
 .22691093+01    .16369351+01    .23658605+01    .14776137+01    .17426505+01    .26507343+01

     STD. ERROR OF EACH OBSERVED Y SUB E OR OF EACH (Y SUB E - YHAT SUB E) AT POST EXPERIMENTAL X(I) OR P(I),
     ASSUMING THESE Y(I) ARE NEWLY OBSERVED INDIVIDUALS WITH WEIGHTS = Q(I)
 .18679677+01    .19859280+01    .21315105+01    .22883288+01    .24507501+01    .26163340+01    .27838593+01
 .29520517+01    .31223104+01    .32925836+01

     T(N-1) AS TEST OF EACH DEVIATION FROM REGRESSION
     -.244          1.492          .796          .598          -.641          -.842

     NORMALIZATION STATISTIC = U =    .95420499+00

     CONFIDENCE LIMITS FOR TRUE Y AT EXPERIMENTAL X(I)
 .32483320+01 ,    .18679471+01 ;    .32483320+01 ,    .18679471+01 ;    .64966639+01 ,    .37358942+01 ;
 .64966639+01 ,    .37358942+01 ;    .64966639+01 ,    .37358942+01 ;    .12993328+02 ,    .74717884+01 ;

     CONFIDENCE LIMITS FOR TRUE Y AT POST EXPERIMENTAL X(I) OR P(I)
 .16241660+02 ,    .93397354+01 ;    .17865826+02 ,    .10273709+02 ;    .19489992+02 ,    .11207683+02 ;
 .21114157+02 ,    .12141656+02 ;    .22738324+02 ,    .13075630+02 ;    .24362489+02 ,    .14009603+02 ;
 .25986656+02 ,    .14943577+02 ;    .27610821+02 ,    .15877550+02 ;    .29234987+02 ,    .16811524+02 ;
 .30859154+02 ,    .17745497+02 ;
```

```
CONFIDENCE LIMITS FOR TRUE Y AT EXPERIMENTAL X(I)
ASSUMING THESE Y(I) ARE NEWLY OBSERVED INDIVIDUALS WITH WEIGHTS = W(I)
.71363582+01 ,   -.20200792+01 ;   .58320097+01 ,   -.71573064+00 ;   .98479999+01 ,   .38455814+00 ;
.80715065+01 ,    .21610516+01 ;   .86015800+01 ,    .16309781+01 ;   .15534027+02 ,   .49310895+01 ;

CONFIDENCE LIMITS FOR TRUE Y AT POST EXPERIMENTAL X(I) OR P(I)
ASSUMING THESE Y(I) ARE NEWLY OBSERVED INDIVIDUALS WITH WEIGHTS = Q(I)
.16526673+02 ,    .90547222+01 ;   .18041623+02 ,    .10097911+02 ;   .19611858+02 ,   .11085816+02 ;
.21204564+02 ,    .12051249+02 ;   .22808477+02 ,    .13005476+02 ;   .24416714+02 ,   .13953378+02 ;
.26032835+02 ,    .14897398+02 ;   .27649469+02 ,    .15838883+02 ;   .29267876+02 ,   .16778635+02 ;
.30887493+02 ,    .17717158+02 ;

EFFECTIVE N

NHAT
.43000000+02    .43000000+02    .10750000+02    .10750000+02    .10750000+02    .26875000+01

NHAT SUB E
.17200000+01    .14214876+01    .11944445+01    .10177515+01    .87755103+00    .76444445+00    .67187500+00
.59515572+00    .53086420+00    .47645430+00

N
.97727273+00    .19111111+01    .91489363+00    .23454545+01    .16862745+01    .72881357+00

N SUB E
.14675766+01    .12984399+01    .11271298+01    .97793952+00    .85261071+00    .74810489+00    .66077603+00
.58738711+00    .52526707+00    .47236231+00

VARY = VARIANCE OF Y WITHIN GROUPS AT ONE X, GROUP SIZE, Y TOTAL

AT X =  .10000000+02    N =  2    SUM Y =  .12000000+02    VARIANCE =  .60000000+01 ;

AT X =  .20000000+02    N =  3    SUM Y =  .33000000+02    VARIANCE =  .37500000+01 ;

WITHIN GROUPS DEGREES OF FREEDOM AND SUM OF SQUARES  *  BETWEEN GROUPS DEGREES OF FREEDOM AND SUM OF SQUARES
                                                     *
                 3               .13500000+02        *                   1                .30000000+02
                                                     *
```

ANALYSIS OF VARIANCE

SOURCE OF VARIATION	DEGREES OF FREEDOM	SUM OF SQUARES	MEAN SQUARE	F
SLOPE OF THE LINE	1	.28139535+03	.28139535+03	.54950045+02
DEVIATIONS FROM THE LINE	5	.25604651+02	.51209302+01	
GROUP MEANS FROM LINE	2	.12104651+02	.60523255+01	.13449612+01
WITHIN GROUPS	3	.13500000+02	.45000000+01	
TOTALS	6	.30700000+03	.51166667+02	